Topic 12

Differentiation

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 28

Q1

The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 1 is 4 marks)

Q2

Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$

(Total for Question 2 is 3 marks)

Q3

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(Total for Question 3 is 5 marks)

Q4

Prove, from first principles, that the derivative of x^3 is $3x^2$

(Total for Question 4 is 4 marks)

Q5

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ giving each term in its simplest form.

(4)

(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(2)

(Total for Question 5 is 6 marks)

Q6

Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

(Total for Question 6 is 6 marks)

End of Questions

Bronze Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
			(4 marks)

Notes

M1 : Differentiation implied by one correct term
A1 : Correct differentiation
M1 : Attempts to substitute $x = 5$ into their derived function
A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

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Q2.

Question number	Scheme	Marks
	$x^4 \to kx^3$ or $x^{\frac{1}{3}} \to kx^{-\frac{2}{3}}$ or $3 \to 0$ (k a non-zero constant)	M1
	$\left(\frac{dy}{dx}=\right)$ $4x^3$.......... , with '3' differentiated to zero (or 'vanishing')	A1
	$\left(\frac{dy}{dx}=\right)$ .... $+\frac{1}{3}x^{-\frac{2}{3}}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3\left(\sqrt[3]{x}\right)^2}$	A1
		[3]
	1st A1 requires $4x^3$, <u>and</u> 3 differentiated to zero. Having '$+C$' loses the 1st A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-\frac{2}{3}}$ loses the 2nd A mark.	

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Q3.

Question	Scheme	Marks	AOs
	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.
Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$
It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Q4.

Question	Scheme	Marks	AOs
	Considers $\frac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3=x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0$, $3x^2+3xh+h^2\to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
		(4 marks)	

B1: Gives the correct fraction for the gradient of the chord either $\frac{(x+h)^3-x^3}{h}$ or $\frac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\frac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x)$, $\frac{dy}{dx}$, y' should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x)\underset{\lim h\to 0}{=}\frac{(x+h)^3-x^3}{h}=3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x)\underset{\lim h\to 0}{=}3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0\Rightarrow f'(x)=3x^2+3x\times 0+0^2=3x^2$

Alternative: B1: Considers $\frac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\frac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

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Q5.

Question Number	Scheme	Marks
	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$	
(a)	$\left\{\frac{dy}{dx} =\right\}$ $5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$	M1
	$= 15x^2 - 8x^{\frac{1}{3}} + 2$	A1 A1 A1
		[4]
(b)	$\left\{\frac{d^2y}{dx^2} =\right\}$ $30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1
		[2] 6
	Notes	
(a)	**M1**: for an attempt to differentiate $x^n \to x^{n-1}$ to one of the first three terms of $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$. So seeing either $5x^3 \to \pm\lambda x^2$ or $-6x^{\frac{4}{3}} \to \pm\mu x^{\frac{1}{3}}$ or $2x \to 2$ is M1. **1st A1**: for $15x^2$ only. **2nd A1**: for $-8x^{\frac{1}{3}}$ or $-8\sqrt[3]{x}$ only. **3rd A1**: for $+2$ ($+c$ included in part (a) loses this mark). **Note:** $2x^0$ is A0 unless simplified to 2.	
(b)	**M1**: For differentiating $\frac{dy}{dx}$ again to give **either** • a correct follow through differentiation of their x^2 term • or for $\pm\alpha x^{\frac{1}{3}} \to \pm\beta x^{-\frac{2}{3}}$. **A1**: for any *correct* expression *on the same line* (accept un-simplified coefficients). For powers: $30x^{2-1} - \frac{8}{3}x^{\frac{1}{3}-1}$ is A0, but writing powers as one term eg: $(15\times 2x) - \frac{8}{3}x^{-\frac{4}{6}}$ is ok for A1. **Note:** Candidates leaving their answers as $\left\{\frac{dy}{dx} =\right\} 15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$ and $\left(\frac{d^2y}{dx^2} =\right)$ $30x - \frac{24}{9}x^{-\frac{2}{3}}$ are awarded M1A1A0A1 in part (a) and M1A1 in part (b). **Be careful:** $30x - \frac{8}{3}x^{-\frac{1}{3}}$ will be A0. **Note:** For an extra term appearing in part (b) on the same line, ie $30x - \frac{8}{3}x^{-\frac{2}{3}} + 2$ is M1A0 **Note:** If a candidate writes in part (a) $15x^2 - 8x^{\frac{1}{3}} + 2 + c$ and in part (b) $30x - \frac{8}{3}x^{-\frac{2}{3}} + c$ then award (a) M1A1A1A0 (b) M1A1	

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Q6.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	$2 - 16x^{-3} = 0$ so $x^{-3} =$ or $x^3 =$, or $2 - 16x^{-3} = 0$ so $x = 2$	M1
	$x = 2$ only (after correct derivative)	A1
	$y = 2 \times "2" + 3 + \frac{8}{"2^2"}$	M1
	$= 9$	A1
		(6)
		Total 6

	Notes for Question
	1[st] M1: At least one term **differentiated (not integrated)** correctly, so $2x \to 2$, or $\frac{8}{x^2} \to -16x^{-3}$, or $3 \to 0$ A1: This answer or equivalent e.g. $2 - \frac{16}{x^3}$ 2[nd] M1: Sets $\frac{dy}{dx}$ to 0, and solves to give x^3 = value or x^{-3} = value (or states x =2 with no working following correctly stated $2 - 16x^{-3} = 0$) A1: $x = 2$ cso (if x = -2 is included this is A0 here) 3[rd] M1: Attempts to substitutes **their positive** x (found from attempt to differentiate) into $y = 2x + 3 + \frac{8}{x^2}$, $x > 0$ Or may be implied by $y = 9$ or correct follow through from their positive x A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)

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Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

The curve C has equation

$$y = 2x - 8\sqrt{x} + 5 \,,\ x \geq 0$$

(a) Find $\dfrac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\dfrac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

(Total for Question 1 is 7 marks)

Q2

The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

(Total for Question 2 is 9 marks)

Contents

Q3

Prove, from first principles, that the derivative of $3x^2$ is $6x$

(Total for Question 3 is 4 marks)

Q4

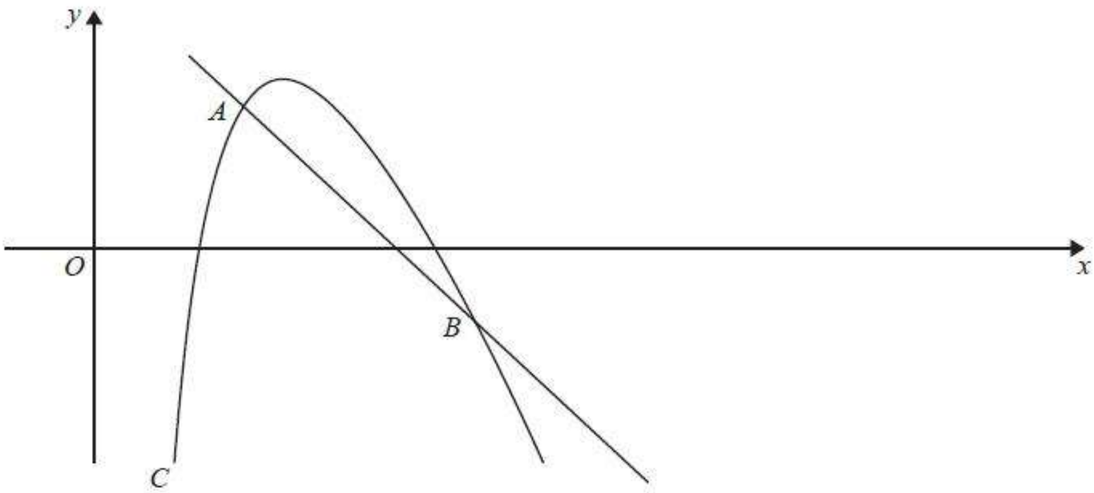

Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \quad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2

Show that the equation of the normal to C at A is $y = -2x + 7$

(Total for Question 4 is 6 marks)

Q5

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \qquad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(Total for Question 5 is 8 marks)

Contents

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2(\frac{1}{4}) - 8\sqrt{(\frac{1}{4})} + 5$ so) $\quad y = \frac{3}{2}$	B1
	(gradient $= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{(\frac{1}{4})}}$ $\{= -6\}$	M1
	Either : $y - "\frac{3}{2}" = "-6"(x - \frac{1}{4})$ **or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"(\frac{1}{4}) + c \Rightarrow c = "3"$	dM1
	So $\underline{y = -6x + 3}$	A1
		[4]
		7 Marks
	Notes	
(a)	**M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once so $x^1 \to 1$ or x^0 or $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ not just $5 \to 0$	
	A1: Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified.	
	A1: $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$	
(b)	**B1:** Obtaining $y = 3/2$ or fractional or decimal equivalent (no working need be seen)	
	M1: An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish **gradient**. This may be implied by -6 or $m = -6$ but not $y = -6$. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$	
	dM1: This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their **tangent** gradient and their value for y_1 (obtained from $x = ¼$, allow slip) i.e.	
	$y - y_1 = m_T(x - \frac{1}{4})$ with their tangent gradient and their y_1	
	or uses $y = mx + c$ with $(\frac{1}{4}$, their $y_1)$ and their tangent gradient	
	Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4	

Contents

Q2.

Question Number	Scheme		Marks
	$y = 6 - 3x - \dfrac{4}{x^3}$		
(a)	$\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4} \text{ or } -3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ $(x^1 \to x^0 \text{ or } x^{-3} \to x^{-4} \text{ or } 6 \to 0)$ A1: Correct derivative	M1 A1
	$\dfrac{dy}{dx} = 0 \Rightarrow -3 + \dfrac{12}{x^4} = 0 \Rightarrow x = ... $ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4} = 0 \Rightarrow \dfrac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\dfrac{d^2y}{dx^2} = \dfrac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

Q3.

Question	Scheme	Marks	AOs
	Considers $\frac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2+6xh+3h^2$	M1	1.1b
	so gradient $=\frac{6xh+3h^2}{h}=6x+3h$ or $\frac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h\to 0$, gradient $\to 6x$ so in the limit derivative $=6x$ *	A1*	2.5
		(4 marks)	

Notes

B1: gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2 = 3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

Contents

Q4.

Question Number	Scheme		Marks
(a)	Substitutes $x = 2$ into $y = 20 - 4\times2 - \frac{18}{2}$ and gets 3		B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$		M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)		dM1
	Method 1 States or uses $y - 3 = -2(x-2)$ or $y = -2x + c$ with their (2, 3)	Method 2 Or: Check that (2, 3) lies on the line $y = -2x + 7$	dM1
	to deduce that $y = -2x + 7$ *	Deduce equation of normal as it has the **same gradient** and passes through **a common point**	A1*
			(6)

Contents

Q5.

Question Number	Scheme		Marks
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}\ x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3},\ V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$	Substitute candidate's value of x where $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
	Notes		
(a)	1st M1 for a three term cubic in the form $\pm\alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm\alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1st M1. 1st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm\lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2nd A1 for $100 - 80x + 12x^2$, **cao**. **Note:** See appendix for those candidates who apply the product rule of differentiation.		

Contents

Gold Questions

Calculators may not be used

The total mark for this section is 33

Q1

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10$, $x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

(2)

(c) State the nature of the turning point.

(1)

(Total for Question 1 is 10 marks)

Q2

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

(Total for Question 2 is 9 marks)

Contents

Q3

$$y = x^2 - k\sqrt{x}\,, \text{ where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

(Total for Question 3 is 4 marks)

Q4

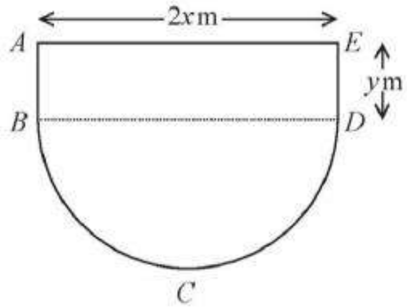

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\dfrac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer in exact form.

(4)

(Total for Question 4 is 10 marks)

Contents

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10\right]$	
	$[y' =] \quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x = \quad ,\frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]
(a)	1^{st} M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2^{nd} M1 for forming a suitable equation using their $y' = 0$ 3^{rd} M1 for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4^{th} M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1	
(b)	M1 for differentiating their y' again A1 should be simplified	
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)	

Q2.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}=\right)-4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2 \Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks <u>could</u> be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2x$ (*)	M1 A1cso
		(6)
(b)	Gradient of normal $=\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1
		(3)
		[9]
(a)	1st M1 for 4 or $8x^{-2}$ (ignore the signs). 1st A1 for both terms correct (including signs). 2nd M1 for substituting $x=2$ into their $\frac{dy}{dx}$ (must be different from their y) B1 for $y_P=-3$, but not if clearly found from the given equation of the <u>tangent</u>. 3rd M1 for attempt to find the equation of tangent at P, follow through their m and y_P. Apply general principles for straight line equations (see end of scheme). <u>NO DIFFERENTIATION ATTEMPTED</u>: Just assuming $m=-2$ at this stage is M0 2nd A1cso for correct work leading to printed answer (allow equivalents with $2x$, y, and 1 terms… such as $2x+y-1=0$).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their m, but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent. M1 for an attempt to find normal at P using their changed gradient and their y_P. Apply general principles for straight line equations (see end of scheme). A1 for any correct form as specified above (correct answer only).	

Contents

Q3.

Question Number	Scheme	Marks
	(a) $\left(\frac{dy}{dx}=\right)\ 2x-\frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$)	M1
	$8-\frac{k}{4}<0$ $\quad k>32$ (or $32<k$) <u>Correct inequality needed</u>	A1 (2) **4**
	(a) M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes <u>called</u> y, and in this case the M mark can be given. $\frac{dy}{dx}=0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. <u>Working</u> must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

Contents

Q4.

Question	Scheme	Marks	AOs
(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\dfrac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \dfrac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give	A1	1.1b
	perimeter = $(4+\pi)\sqrt{\left(\frac{500}{4+\pi}\right)}$ m.	(4)	
			(10 marks)

Notes

(a) B1 : Correct area equation
M1 : Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P
M1 : Use correct equation for perimeter with their y substituted
A1*: Completely correct solution to obtain and state printed answer
(b) M1 : States $x>0$ and $y>0$ and uses their expression from (a) to form inequality
A1*: Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly.
(c) M1: Attempt to differentiate P (deals with negative power of x correctly)
A1 : Correct differentiation
M1 : Sets derived function equal to zero and obtains $x=$
A1: The value of x may not be seen (it is $\sqrt{\left(\frac{500}{4+\pi}\right)}$).
Need to see $(4+\pi)\sqrt{\left(\frac{500}{4+\pi}\right)}$ with units included for the perimeter.

Contents

Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

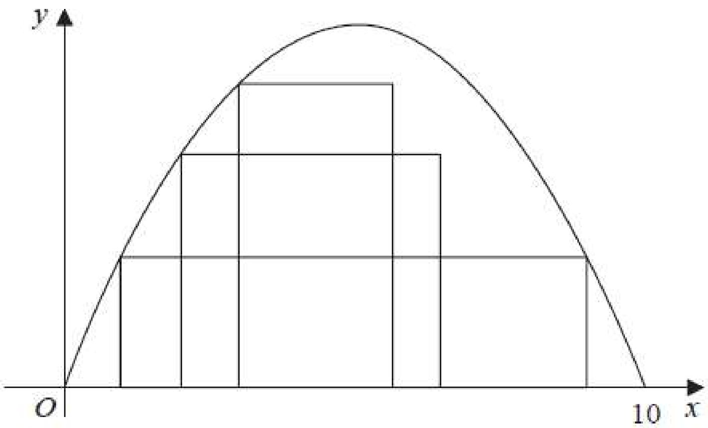

Figure 2

Figure 2 shows a sketch of the parabola with equation y $y=\frac{1}{2}x(10-x), 0 \leq x \leq 10$

This question concerns rectangles that lie under the parabola in the first quadrant. The bottom edge of each rectangle lies along the x-axis and the top left vertex lies on the parabola. Some examples are shown in Figure 2.

Let the x coordinate of the top left vertex be a.

(a) Explain why the width, w, of such a rectangle must satisfy $w \leq 10 - 2a$

(2)

(b) Find the value of a that gives the maximum area for such a rectangle.

(5)

Given that the rectangle must be a square,

(c) find the value of a that gives the maximum area for such a square.

(3)

Given that the area of the rectangles is fixed as 36

(d) find the range of possible values for a

(6)

(+S1)

(Total for Question 7 is 17 marks)

Contents

Platinum Mark Scheme

1(a)	The rectangle must lie under the parabola, so maximum width will occur when the top right vertex also lies on the parabola. Ie recognises the symmetry and forms an equation. Allow a suitable sketch as evidence.		**M1**	1
	By symmetry about the line $x=5$, this occurs at $\left(10-a,\frac{1}{2}a(10-a)\right)$, hence width satisfies $w \leqslant 10-a-a=10-2a$* Must be convincing reason.		**A1***	2
			(2)	
(b)	Maximum area must occur for a full width rectangle, ie when $w=10-2a$		**B1**	2
	Thus max area occurs for $A=\dfrac{1}{2}a(10-a)\times(10-2a)$		**M1**	3
	Attempts $\dfrac{\mathrm{d}A}{\mathrm{d}a}=\dfrac{1}{2}(10-2a)\times(10-2a)+\dfrac{1}{2}(10a-a^2)\times-2\left(=3a^2-30a+50\right)$ and sets $\dfrac{\mathrm{d}A}{\mathrm{d}a}=0$ and attempts to find a		**M1**	3
	$\Rightarrow 3(a-5)^2-3\times25+50=0\Rightarrow a=5\pm\sqrt{\dfrac{25}{3}}=5\pm\dfrac{5}{\sqrt{3}}$	Any correct method to solve the quadratic.	**M1**	3
	(But need $0<a<5$ to give a valid rectangle and as area is zero at either end of this interval so)		**(S+)**	
	max area occurs when $a=5-\dfrac{5\sqrt{3}}{3}$ (oe simplified)		**A1**	2
			(5)	
(c)	Max square area needs $10-2a=\dfrac{1}{2}a(10-a)\Rightarrow a=\ldots$	Sets up correct equation.	**M1**	3
	$20-4a=10a-a^2\Rightarrow a^2-14a+20=(a-7)^2-49+20=0$ $\Rightarrow a=7\pm\sqrt{29}$	Solves the quadratic, any valid means.	**dM1**	3
	But need $0<a<5$ (and $\sqrt{29}<7$) so $a=7-\sqrt{29}$	Selects correct root.	**(S+) A1***	3
			(3)	
(d)	If area is 36, then width is given by $w=\dfrac{36}{\frac{1}{2}a(10-a)}=\dfrac{72}{10a-a^2}$ (oe) Therefore need solutions to $\dfrac{72}{10a-a^2}\leqslant 10-2a$ OR need solutions to $\dfrac{1}{2}\left(a+\dfrac{72}{10a-a^2}\right)\left(10-\left(a+\dfrac{72}{10a-a^2}\right)\right)\leqslant\dfrac{1}{2}a(10-a)$ or other valid inequality		**B1**	1

	in a set up e.g. $10a-a^2\backslash\ldots 10b-b^2 \Rightarrow (b-a)(b+a)+10(a-b)\ldots 0$ $\Rightarrow 10-(a+b)\ldots 0$ (as $a \neq b$) followed by substitution of $b = a+\dfrac{72}{10a-a^2}$ This mark is for a correct reasoning of the required inequality, If no reason is given and equation is it is B0, but all other marks are possible.		
	Forms a suitable cubic using the maximum width and height (may be equation or inequation.	**M1**	3
	$\Rightarrow a^3-15a^2+50a-36\ldots 0$ — Correct cubic achieved as equation or inequation.	**A1**	3
	Identifies $(a-1)$ as factor (factor theorem) and attempts to factorise $\Rightarrow (a-1)\left(a^2-14a+36\right)\ldots 0$	**M1**	3
	$a^2-14a+36=(a-7)^2-49+36 \Rightarrow$ CVs are $a=1, 7\pm\sqrt{13}$ — Finds CVs	**M1**	3
	(positive cubic with roots $1<7-\sqrt{13}\,(<5)<7+\sqrt{13}$ (as $3<\sqrt{13}<4$)) So possible values of a are $1,, \ a ,, \backslash 7-\sqrt{13}$	**(S+)** **A1**	2
		(6)	
S1	S1 mark: Award S1 for a clear and concise solution that is - fully correct with no S- point or - that scores 13+ and includes an S+ point and no S-.	**(1)**	2
			(16 + 1 marks)

Notes:

(b) S+ for explaining clearly why the root outside $0<a<5$ is rejected.
(c) S+ for justifying the root lies in acceptable domain for a.
(c) S- for a cumbersome strategy. **S+** for justification of roots/which are in valid domain.